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Seeds of Large-Scale Anisotropy in Pre-Big-Bang Cosmology

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Within a string cosmology context, the large-scale temperature anisotropies may arise from the contribution of seeds to the metric fluctuations. We study the cases of electromagnetic and axion seeds. We find that massless or very light axions can lead to a flat or slightly tilted blue spectrum that fits current data.

1. INTRODUCTION

I will briefly present some results[1, 2] on the seeds of large-scale anisotropy in the context of string cosmology. I work on the pre-big-bang scenario (PBB) [3], defined as a particular model of inflation inspired by the duality properties of string theory. The question which I address is whether we can reproduce the observed amplitude and slope of the large-scale temperature anisotropy and of large-scale density perturbations within the PBB scenario.

First-order scalar and tensor metric perturbations lead to primordial spectra that grow with frequency [4] with a normalization imposed by the string cutoff at the shortest amplified scales. These blue spectra have too little power at scales relevant for the observed anisotropies in the cosmic microwave background radiation (CMBR). In contrast, the axion energy spectra were found to be logarithmically diverging, leading to red spectra of CMBR anisotropies which are in conflict with observations. These results already ruled out four-dimensional PBB cosmology.

However, if one allows for internal contracting dimensions in addition to the three expanding ones, the supersymmetric partner of the dilaton (the universal axion of string theory) can lead to a flat Harrison–Zel'dovich (HZ)

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spectrum of fluctuations for an appropriate relative evolution of the external and the compactified internal dimensions [5]. Thus, the PBB scenario may contain a natural mechanism for generating large-scale anisotropy via the seed mechanism [6] (i.e., fluctuations of one component of the energy momentum tensor can feed back on the metric through Einstein's equations).

In what follows, I consider the possibility that vacuum fluctuations of the electromagnetic and of the axion field may act, at second order, as scalar seeds of large-scale structure and CMBR anisotropies. The induced perturbations are isocurvature perturbations. More precisely, I examine whether the metric perturbation spectrum triggered by these seeds can be flat enough to match present measurements, consistent with the COBE normalization of the amplitude on large scales, and with the normalization imposed by the string cutoff at the shortest amplified scales.

2. LARGE-SCALE PERTURBATIONS IN THE PRESENCE OF SEEDS

I will derive a general formula for large-scale CMBR anisotropies in models where perturbations are triggered by seeds. I consider the case of scalar perturbations.

2.1. Cosmological Perturbation Theory with Seeds

We express the Fourier components of the energy momentum tensor of the seeds $T_{\mu\nu}$ in terms of four scalar "seed functions" f_ρ, f_p, f_v, and f_π [7]:

$$T_{00}\,(\mathbf{k},\eta) = M^2 f_\rho(\mathbf{k},\eta) \tag{1}$$

$$T_{j0}(\mathbf{k},\eta) = -iM^2\, k_j f_v\,(\mathbf{k},\eta) \tag{2}$$

$$T_{ij}(\mathbf{k},\eta) = M^2\left[\left(f_p(\mathbf{k},\eta) + \frac{k^2}{3} f_\pi(\mathbf{k},\eta)\right)\gamma_{ij} - k_i k_j f_\pi(\mathbf{k},\eta)\right] \tag{3}$$

Here M denotes an arbitrary mass scale, introduced for dimensional reasons; η denotes conformal time, and γ represents a metric of constant curvature.

The perturbed Einstein equations read [7]

$$k^2\Phi = 4\pi G\rho a^2 D + \epsilon[f_\rho + 3(\dot{a}/a)f_v] \tag{4}$$

$$\Phi + \Psi = -8\pi G a^2 k^{-2}\, p\Pi - 2\epsilon f_\pi \tag{5}$$

where $\epsilon \equiv 4\pi G M^2$, a is the scale factor, and dot denotes derivative with respect to η. Here Π is the anisotropic stress potential, V is the peculiar velocity potential, D (and D_g, which I will use later) is a gauge-invariant density perturbation variable, and Φ, Ψ are two geometric quantities, called the Bardeen potentials. Since large-scale CMBR anisotropies are induced at recombination and later, we set $\Pi = 0$.

The large-scale anisotropies of CMBR are determined by the combination $\Psi - \Phi$:

$$\Psi - \Phi \sim \max \left\{ \epsilon f_\pi, \epsilon\eta^2 \left(f_\rho + 3\frac{\dot{a}}{a} f_v \right) \right\} \tag{6}$$

2.2. The Seed Contribution to CMBR Anisotropies

I calculate the CMBR anisotropies and their contribution to $\Delta T/T$ via the Sachs–Wolfe effect [8]. The temperature perturbation reads [7]

$$\frac{\delta T(\mathbf{n})}{T} = \left[\frac{1}{4} D_g + V_j n^j + \Psi - \Phi \right] (\eta_{\rm dec}, \mathbf{x}_{\rm dec}) + \int_{\eta_{\rm dec}}^{\eta_0} (\dot{\Psi} - \dot{\Phi})\, (\eta, \mathbf{x}(\eta)) d\eta \tag{7}$$

where $\mathbf{x}(\eta) = \mathbf{x}_0 - (\eta_0 - \eta)\mathbf{n}$ is the unperturbed photon position at time η for an observer at $\mathbf{x}_0$, η_0 is the conformal time today, and $\mathbf{x}_{\rm dec} = \mathbf{x}(\eta_{\rm dec})$.

The angular power spectrum of CMBR anisotropies is expressed in terms of the dimensionless coefficients C_ℓ which appear in the expansion of the angular correlation function in terms of the Legendre polynomials P_ℓ:

$$\left\langle \frac{\delta T}{T} (\mathbf{n}) \frac{\delta T}{T} (\mathbf{n}') \right\rangle_{(\mathbf{n}\cdot\mathbf{n}' = \cos\vartheta)} = \frac{1}{4\pi} \sum_l (2\ell + 1) C_\ell P_{\ell(\cos\vartheta)} \tag{8}$$

Here the brackets denote spatial average, or expectation values if perturbations are quantized. To determine the C_ℓ we Fourier-transform Eq. (7), defining

$$\varphi(\mathbf{k}) = \frac{1}{\sqrt{V}} \int_V \varphi(\mathbf{x}) e^{i\mathbf{k}\cdot\mathbf{x}} d^3x \tag{9}$$

For the coefficients C_ℓ of Eq. (8) we obtain

$$C_\ell = \frac{2}{\pi} \int \frac{\langle |\Delta_\ell(\mathbf{k})|^2 \rangle}{(2\ell + 1)^2} k^2 dk \tag{10}$$

where

$$\frac{\Delta_\ell}{2\ell + 1} = \frac{1}{4} D_g(\mathbf{k}, \eta_{\rm dec}) j_\ell(k\eta_0) - j'_\ell(k\eta_0) \mathbf{V}(\mathbf{k}, \eta_{\rm dec}) + k \int_{\eta_{\rm dec}}^{\eta_0} (\Psi - \Phi)(\mathbf{k}, \eta') j'_\ell(k\eta_0 - k\eta') d\eta' \tag{11}$$

and j'_ℓ stands for the derivative of j_ℓ with respect to its argument. On large angular scales, $k\eta_{\rm dec} \ll 1$, the SW contribution dominates and we obtain.[2]

$$C_\ell^{\rm SW} = \frac{2}{\pi} \int k^4 dk \left\langle \left[\int_{\eta_{\rm dec}}^{\eta_0} (\Psi - \Phi)(\mathbf{k}, \eta) j'_\ell\,(k\eta_0 - k\eta) d\eta \right]^2 \right\rangle \tag{12}$$

We approximate [2] the Bardeen potentials Ψ, Φ on superhorizon scales by a power-law spectrum: $\langle|\Psi - \Phi|^2\rangle = C^2(k)\,(k\eta)^{2\gamma}$. Furthermore, we consider[2] models where the seed contribution does not grow in time on subhorizon scales. Thus,

$$\Psi - \Phi \approx \begin{cases} C(k)(k\eta)^\gamma, & k\eta \ll 1 \\ C(k), & k\eta \gg 1 \end{cases} \tag{13}$$

We further assume [2] that also $C(k)$ is given by a simple power law. Thus, we have

$$C(k) = \begin{cases} \mathcal{N} k^{-3/2}(k/k_1)^\alpha, & k \leq k_1 \\ 0, & k > k_1 \end{cases} \tag{14}$$

where $\mathcal{N}$ is a dimensionless constant, and k_1 denotes a comoving cutoff scale. Inserting Eq. (14) in Eq. (12), we obtain [2]

$$C_\ell^{\rm SW} \approx \mathcal{N}^2 \frac{2}{\pi} \int_0^{k_1} \frac{dk}{k} \left(\frac{k}{k_1}\right)^{2\alpha} |I(k)|^2 \tag{15}$$

where

$$I(k) = \int_{(k\eta)_{\rm dec}}^{1} d(k\eta)(k\eta)^\gamma\, j'_\ell(k\eta_0 - k\eta) + j_\ell(k\eta_0 - 1) \tag{16}$$

We compare $C_\ell^{\rm SW}$ with the inflationary result: $C_\ell^{\rm SW} \propto \Gamma(\ell - 1/2 + n/2)/\Gamma(\ell + 5/2 - n/2)$, where n denotes the spectral index. The scale-invariant spectrum, as found by the DMR experiment [9], requires $0.9 \leq n \leq 1.5$. Thus, we get [2]

$$-0.05 < \gamma + 1 + \alpha < 0.25, \qquad \gamma \leq -1, \qquad n \simeq 3 + 2(\alpha + \gamma) \tag{17}$$

$$-0.05 < \alpha < 0.25, \qquad \gamma > -1, \qquad n = 1 + 2\alpha \tag{18}$$

3. SEEDS FROM STRING COSMOLOGY

In this section I compute the seed functions f_ρ, f_v, f_π and estimate the Bardeen potentials for electromagnetic and axion perturbations.

3.1. Electromagnetic Seeds

Consider a stochastic background obtained by amplifying the quantum electromagnetic fluctuations of the vacuum. For purely magnetic seeds (the electric component of the stochastic background is rapidly dissipated, due to the conductivity of the cosmic plasma), on superhorizon scales we obtain $f_v = 0$, $f_\pi \gg \eta^2 f_\rho$, leading to [2]

$$k^3|\Psi - \Phi|^2 (k, \eta) \approx \mathcal{N}^2(k\eta)^{2\gamma} (k/k_1)^{2\alpha} \tag{19}$$

with

$$\gamma = \begin{cases} -4, & \mu \leq 3/4 \\ 2\mu - 11/2, & 3/4 \leq \mu \leq 3/2 \end{cases} \tag{20}$$

$$\alpha = \begin{cases} 7/2, & \mu \leq 3/4 \\ 5 - 2\mu, & 3/4 \leq \mu \leq 3/2 \end{cases} \tag{21}$$

$$\mathcal{N} = \left(\frac{H_1/M_p}{4\pi}\right)^2 (k_1\eta_{eq})^2 \qquad \text{in both cases} \tag{22}$$

($\mu < 3/2$ to avoid photon overproduction). H_1 is the physical cutoff scale at which the universe becomes immediately radiation-dominated, and M_p is the Planck mass.

Since in both cases $\gamma + 1 < 0$, the seeds decay fast enough outside the horizon. However, in both cases $\gamma + \alpha = -0.5$, which implies $n = 2$. Such a spectrum grows too fast with frequency to fit the COBE measurements. The quadrupole amplitude [10] $Q_{\text{rms-PS}} = \sqrt{(5/4\pi)C_2}T_0 = (18 \pm 2)$ μK leads to $C_2 = (1.09 \pm 0.23) \times 10^{-10}$. Thus, compatibility with the COBE normalization implies [2]

$$(6 - \alpha) \log_{10} (H_1/M_p) \lesssim 55(\alpha - 2) - 6 + \log_{10}(\gamma + 1)^2 \tag{23}$$

This constraint is easily satisfied by a growing seed spectrum, $\alpha > 2$. Even in the limiting case $\alpha = 2$, this condition is marginally compatible even with the maximal expected value $H_1 \sim M_s \sim 5 \times 10^{17}$ GeV.

3.2. Axionic Seeds

We consider pseudoscalar vacuum fluctuations amplified by the time evolution of a higher dimensional background. We first consider massless axions. If $\mu < 3/4$, the situation is like that for electromagnetic seeds. The induced CMBR fluctuations have the wrong spectrum, but their amplitude is sufficiently low to avoid conflict with observations. However, if $3/4 \leq \mu \leq 3/2$, we obtain [1,2]

$$k^3|\Psi - \Phi|^2(k, \eta) \approx \mathcal{N}^2(k\eta)^{2\gamma}\,(k/k_1)^{2\alpha}, \qquad \gamma = 2\mu - 7/2, \qquad \alpha = -2\mu + 3 \tag{24}$$

For $\mu = 3/2$ we obtain a Harrison–Zel'dovich spectrum with amplitude $\mathcal{N} \simeq (H_1/M_p)^2$. The nonconformal coupling of the axions to the metric leads to an additional amplification of perturbations after the matter–radiation equality. The normalization of the axion spectrum to the COBE amplitude imposes the constraint[1, 2]

$$\log_{10}\frac{H_1}{M_p} \simeq \frac{164 - 116\mu}{1 + 2\mu} \qquad \text{with} \quad 1.4 < \mu < 1.5 \tag{25}$$

implying

$$3 \times 10^{-3} \lesssim H_1/M_p \lesssim 2.6 \tag{26}$$

This condition is perfectly compatible with $H_1 \sim M_s \sim 5 \times 10^{17}$ GeV.

Let us now turn to the case of massive axions. In this case, the f_π contribution to Φ, Ψ is negligible when the superhorizon modes are already nonrelativistic at the time of decoupling, and we obtain [2] constant Bardeen potentials with

$$\gamma = 0, \qquad \alpha = 3 - 2\mu, \qquad \mathcal{N} = (H_1/M_p)^2(m/H_{\rm eq})^{1/2} \tag{27}$$

where m denotes the axion mass. For $\mu = 3/2$ we obtain a flat Harrison–Zel'dovich spectrum. The amplitude of perturbations is enhanced by the factor $(m/H_{\rm eq})^{1/2}$. Thus, the axion mass m has to be bounded to avoid conflicting with the COBE normalization $C_2 \approx 10^{-10}$. In addition, we impose $1.4 < \mu < 1.5$ and $m > H_{\rm sec} \sim H_{\rm eq}$, and we require that the present axion energy density is constrained by the critical energy density. We find that for a typical scale $H_1 \sim M_s \sim (10^{-1}–10^{-2})M_p$, the maximal allowed axion mass window is [2]

$$10^{-27}\ \text{eV} \lesssim m \lesssim 10^{-17}\ \text{eV} \tag{28}$$

4. CONCLUSIONS

I have briefly discussed, in the context of the PBB scenario, the possibility that the large-scale temperature anisotropies may arise from the contribution of seeds to the metric fluctuations. In particular, I considered the cases in which the seed inhomogeneity spectrum is due to vacuum fluctuations of the electromagnetic field and of the (Kalb–Ramond) axion field

In the first case, I showed that electromagnetic fluctuations lead to a spectrum that grows too fast with frequency to be compatible with COBE observations. Since the contribution of electromagnetic seeds to the large-scale anisotropy is negligible, there are no constraints from the COBE normalization to the production of seeds for generating the galactic magnetic fields via the amplification of electromagnetic vacuum fluctuations due to a dynamical dilaton background [12].

In the second case, I discussed how a stochastic background of massless axions, produced within the context of the PBB scenario, is a possible candidate for an explanation of the large-scale anisotropy measured by COBE satellite. Regarding massive axions, I showed that if the axion mass is such that all modes outside the horizon at decoupling are already nonrelativistic, then a slightly tilted blue spectrum is still compatible with the amplitude and slope measured by the COBE satellite, provided the axion mass is inside an appropriate window, in the ultralight mass region.

As a next step, one has to study the predictions of this model regarding the acoustic peaks in the CMBR anisotropy power spectrum and the linear dark matter power spectrum and compare them with currently available experimental and observational data. Some preliminary results are discussed in ref. 11. The authors found [11] a strong dependence of their predictions on the overall evolution of extra dimensions during the PBB phase. In other words, further experimental and observational data coming from the CMBR anisotropies and the galaxy distribution may provide some information about the evolution of string theory's extra dimensions.

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